AEC Computing and Applied Mathematics Center

AEC RESEARCH AND DEVELOPMENT REPORT

NYO-2544

SHERWOOD PROGRESS REPORT

No. 3

(July 1958 to July 1959)

H. Grad and J. Berkowitz

July 31, 1959

Institute of Mathematical Sciences

NEW YORK UNIVERSITY

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Personnel

The average number of equivalent full-time scientific staff members was about 16 theoreticians and 2 experimenters. The total number of individuals involved was about 25; these include 13 of faculty rank or with Ph.D.'s (8.5 equivalent full-time) and 12 research assistants (9.5 equivalent full-time).

Propagation of Hydromagnetic Waves

The general problem is the description of the properties of small amplitude signals propagating in a plasma. To this end, the computation of a dispersion relation should be considered to be the beginning rather than the end of the problem. In specific cases which could be evaluated in detail, the properties of plane waves were found to be not only incomplete but even misleading. For example, non-plane fronts can be found which propagate faster in some directions than the maximum speed of propagation of a plane wave in <u>any</u> direction [22-VIII]; similarly, one mode whose plane wave fronts propagate slower than those of another mode in every direction can catch up and pass the second mode if the front is not plane [3]. Complete modes (as distinguished from plane waves) are found to separate only in special cases and, even then, only in a certain subtle sense [3].

The types of question which were approached are the possibility of separation of modes (in arbitrary disturbances rather than plane waves), channeling of energy propagation and radiation patterns, and the general propagation of arbitrary wave fronts including refraction, focussing, etc. with the idea of eventual application to localized plasma diagnostics. The deeper questions force one to study, at least initially, some of the simpler models of plasma behavior. For the classical perfectly conducting fluid, an extensive and almost complete analysis has been made of the propagation of arbitrary

wave fronts into a general inhomogeneous medium (geometrical optics)[18]. Not only the position of the front, but also the variation in amplitude are given by simple formulas. The more difficult problem of general energy and signal transfer (not necessarily related to a wave front) has been successfully attacked in special geometries [3] and is being extended to more general cases by the use of recently refined asymptotic techniques [19]. For perturbations about a constant magnetic field, one mode was found which propagates purely one-dimensionally, guided along the magnetic lines [3]. Conceivably, this could be used as a diagnostic tool to obtain average properties of an open-ended plasma along a single magnetic line. The problem of propagation into a non-uniform field is being studied.

Extension of these results to more realistic plasma models using the Boltzmann equation are under way. Since many of the questions have never been satisfactorily answered for even a simple gas without ionization or electromagnetic effects present, it was found to be necessary to first develop some parts of this simpler theory before dealing with plasmas. As a first step, an asymptotic approximation was obtained to the time-independent fundamental solution of the Boltzmann equation [4]. This is being extended to time-dependent propagation and to similar analysis in plasmas.

Particle Orbits

The proof of the asymptotic validity of the guiding center expansion [27] was submitted for publication [10].

A large number of general results concerning adiabatic invariants and the asymptotic behavior of charged particle orbits was obtained. In September (this was announced informally at the Geneva conference) C. S. Gardner proved the existence of the "second" adiabatic invariant (even to arbitrarily high order) for axially symmetric systems. These results were extended [40a], using classical perturbation theory of Hamiltonian systems to obtain

1) a new and simplified proof of the invariance of the magnetic moment to arbitrarily high order

2) under appropriate circumstances (roughly some sort of mirror arrangement -- but not necessarily axially symmetric) the invariance to arbitrary order of a second adiabatic invariant.

3) under appropriate circumstances the invariance of a third invariant

4) extension of all these results to relativistic particles.

The Hamiltonian treatment yields a reduction of the original sixth order system to fourth order on elimination of the magnetic moment as a "constant" of the motion and a similar reduction to second order on elimination of the second invariant. What remains is a system of one degree of freedom

describing the motion of the "line" to which the particle is "tied". A brief account was submitted for publication [13], and a more detailed version is being prepared for publication.

These adiabatic results do not give much information about deviations from adiabaticity. Some preliminary analytic results in this direction were obtained, but much more work is required.

Special problems in which applications of particle orbit theory arise are described under Shock Theory, Equilibrium and Stability, and Cusped Geometries.

Shock Theory

The analysis of the self-consistent steady shock (non-colliding particles) propagating in a direction perpendicular to the magnetic field was continued. It had previously been observed that there are at least two significantly different sources of entropy production, in particular one originating from particles which are reflected back by their first encounter with the charge separation electric field [1c]. It was now observed that there are parameter values for which this entropy increase is an appreciable part of the maximum which could occur across a complete shock [40c]. Thus, even ignoring other mechanisms, there can be appreciable irreversible heating of the ions in a very thin layer. Specifically,

there are interesting ranges of the physical parameters which yield an essentially discontinuous shock front across which ions alone are irreversibly heated; this front is followed by an approximately adiabatic periodic wave train. A full-scale analysis and computation of the magnitude of this heating as a function of various parameters was carried out and will be issued shortly [11].

Details of some of the earlier work in this subject were issued in [2].

Several fluid-type approximations to this problem were continued (cf. [35g]), but, thus far, unsuccessfully (i.e., in prediction of a damped shock profile).

Transport Theory

By making strong use of symmetry relations, it was found to be possible to elucidate and clarify the jungle of transport coefficients and collision integrals which arise from the Boltzmann equation [14]. The program aimed at improved values for these coefficients, see [29] and [7], was continued. A dictionary of collision integrals is being prepared (L. Sirovich) to avoid future repetitions of the extensive redundant calculations in the literature.

The question of the slowing down and thermalization of a single fast ion was treated, [40d], [12]. Care had to be taken to include both electron and ion effects since either one dominated in different regions. Quite complicated thermalization patterns were found, including development of a secondary peak in the distribution and the transition from an initial δ-function through a distribution much wider than the ultimate Maxwellian before final thermalization. The final decay to Maxwellian was found to be approximately exponential, but the decay time depends strongly on the initial energy of the ion. In other words, reference to a universal ion "thermalization" time can be quite misleading.

The asymptotic evaluation of the tail end of the ion distribution and resultant correction in the thermonuclear reaction rate [28], [1e], was improved to obtain an exact rather than an asymptotic result (R. Liboff - numerical results not yet complete).

Two questions of anomalous resistivity induced by plasma oscillations were considered. The first depends on the increased dispersion of a particle due to the presence of externally induced plasma oscillations which are only slowly damped [40b]. This effect can be exceptionally large in a sheath. The second results from the introduction of plasma oscillations into the formulation of Ohm's law, thereby modifying the diffusion rate of plasma into field (J. Thurber, not yet complete). These

two effects are independent; in particular, the plasma resistivity value to be used in the second problem should be the corrected value as given by the first.

Equilibrium and Stability

The problem of the self-consistent equilibrium of a mirror-machine geometry in the guiding center limit (with non-diagonal stress tensor, generalizing the fluid model, $\nabla p = J \times B$) was solved in certain limiting cases (A. Oppenheim - report to appear). More general solutions also seem to be accessible and are being studied. A stability analysis will follow.

Existence theorems have been proved for a variety of axially symmetric and toroidal free boundary equilibria [40e] (M.Schechter and C. Morawetz - detailed report to appear).

The problem of the stability of a toroidal "stabilized" pinch has been formulated and prepared for computation (R. D. Richtmyer, R. Lüst, B. Suydam).

A stability analysis using the guiding center fluid equations (with non-diagonal stress tensor) [34h] was found to yield instability when the ratio of parallel to perpendicular pressure is either too large or too small [40f]. At

second glance, this result is found to be illusory or, at least, hard to interpret since the equations of motion are found to be not well-posed when the stated inequalities are exceeded. In other words, the motion is not unstable; it is nonexistent. This result may cast doubt on all stability analyses which use the guiding center limit since it is this limit which seems to produce the paradoxical result.

Comprehensive and detailed reports covering the material which was briefly summarized in the two Geneva papers [1b] and [1d] will be issued shortly, [8-IX] and [8-X].

Cusped Geometries

Work in this subject consisted mainly in the continuation of problems which have been under study for some time. These include the computation of axially symmetric free boundary configurations and the computation of nonadiabatic orbits in cusped magnetic fields for the improved estimation of particle loss rates.

References: [1a], [25], [6], [9].

Experimental

The experimental program was terminated as of July 1959. A final report is being prepared [15].

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b. Hydromagnetic Equilibria and Force-Free Fields by H. Grad and H. Rubin

c. Hydromagnetic Shock Waves in High-Temperature Plasmas C. S. Gardner, H. Goertzel, H. Grad, C. S. Morawetz, M. H. Rose, H. Rubin

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